E 2211 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2018

Second Semester

Core Course 2—ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(Common for B.Sc. Mathematics Model I, Model II and B.Sc. Computer Application)

[2013 to 2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Write the equation of the normal at the point t on the parabola $y^2 = 4ax$.
2. When we say that two lines are conjugate with respect to a conic ?
3. Write the equation of the asymptotes of the hyperhola $2x^2 + 2xy - 3x + 4y = 0$.
4. Write the polar equation of a circle whose centre is (c, α) and radius a.
5. Write the equation of the line joining the points (r_1, θ_1) and (r_2, θ_2).
6. Show that $\cosh^2 x - \sinh^2 x = 1$.
7. Express sech$x$ in terms of e^x.
8. State Cayley Hamilton theorem.
9. Define adjoint of a matrix.
10. What do you mean by a consistent system of linear equations ?

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Prove that the tangents at the extremities of any focal chord of a parabola intersect at right angles on the directrix.
12. Find the pole of the line $lx + my + n = 0$ with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
13. If the normals of two points of a hyperbola intersect on the curve, show that the product of the ordinates of the points is $8a^2$.
14. Find the co-ordinates of the centre of the circle $r = A \cos\theta + B \sin\theta$.

Turn over

15. Find the condition that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$.

16. Show that $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.

17. If $u = \log\tan\left(\frac{\pi}{4} + \theta/2\right)$, prove that $\cosh u = \sec\theta$.

18. Separate $\cos(x - iy)$ into real and imaginary parts.

19. If $\frac{\sin\theta}{\theta} = \frac{1349}{1350}$, prove that the angle θ is very nearly equal to $\frac{1}{15}$ radian.

20. What are the elementary transformations of a matrix ?

21. Find the characteristic polynomial of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$.

22. Prove that the characteristic roots of a Hermitian matrix are real.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. If the normal at t_1 to the parabola $y^2 = 4ax$ meets the curve again at t_2, show that $t_1^2 + t_1 t_2 + 2 = 0$.

24. Find the locus of the poles of the chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which subtend a right angle at the centre.

25. Prove that the orthocentre of any triangle inscribed in a rectangular hyperbola lies on the curve.

26. Find the equation of the normal at a point on the conic $\frac{l}{r} = 1 + e\cos\theta$, whose vectorial angle is α.

27. Show that the sum of the infinite series $\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1.3}{2.4}\cos 3\theta + \frac{1.3.5}{2.4.6}\cos 4\theta + \ldots$ is $\cos\left(\frac{\pi}{4} + \frac{3\theta}{4}\right)\left(2\sin\frac{\theta}{2}\right)^{-1/2}$.

28. Resolve $x^6 + 1$ into real factors.

29. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$.

30. Verify Cayley-Hamilton theorem for $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

31. Find the eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Show that the locus of the poles of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $y^2 a^6 - x^2 b^6 = (a^2 + b^2)^2 x^2 y^2$.

(b) If e and e_1 are the eccentricities of a hyperbola and its conjugate show that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$.

33. (a) Derive the polar equation of a conic.

(b) Find the equation of the chord of the conic $\frac{l}{r} = 1 + e\cos\theta$ joining the points whose vectorial angles are $\alpha - \beta$ and $\alpha + \beta$.

34. (a) Sum the series $\sinh\alpha - \frac{1}{2}\sinh 2\alpha + \frac{1}{3}\sinh 3\alpha \ldots \infty$.

(b) Resolve into factors the expression $x^{2n} - 2x^n \cos n\theta + 1$.

35. (a) Solve the system of equations :

$$\begin{aligned} x + y + z + w &= 0 \\ x + y + z - w &= 4 \\ x + y - z + w &= -4 \\ x - y + z + w &= 2. \end{aligned}$$

(b) Solve completely the following system of equations :

$$\begin{aligned} x - 2y + z - w &= 0 \\ x + y - 2z + 3w &= 0 \\ 4x + y - 5z + 8w &= 0 \\ 5x - 7y + 2z - w &= 0. \end{aligned}$$

$(2 \times 15 = 30)$